Indian Statistical Institute
Bangalore Centre
Mid-semster Examination
Topology, B. Math II Year
II Semester, 2103-14

Time limit is 3hours. Maximum marks: 100.

All spaces are assumed to be T_1.

1. Consider $\mathbb{R}$ with the cofinite topology. Which of the following sequences are convergent and what are their limits?

a) $\{1, 2, 3, ...\}$

b) $\{1, 1, 2, 1, 3, 1, 4, 1, ...\}$ ($a_n = n$ for n odd, 1 for n even)

c) $\{1, 2, 1, 2, 1, 2, ...\}$ ($a_n = 1$ for n odd, 2 for n even) [5+5+5]

2. Prove that any second countable space is first countable. Is the converse true? Justify. [10+15]

3. Show that $\mathbb{R}$ with the co-finite topology is not regular. Prove also that $\mathbb{R}$ with the co-countable topology is not regular. [5]

4. If x and y are distinct points of a regular space (X, τ) show that there exist open sets U and V such that $x \in U, y \in V$ and $\bar{U} \cap \bar{V} = \emptyset$. [15]

5. Prove that any open connected set in $C[0, 1]$ is polygonally connected.

[Here $C[0, 1]$ is the space of real valued continuous function on $[0, 1]$ with the metric: $d(f, g) = \sup\{|f(x) - g(x)| : 0 \leq x \leq 1\}$; a polygonal path is a path made up of a finite number of line segments]. [20]

6. Let A be a subgroup of $\mathbb{R}$ under addition. Show that either A is dense in $\mathbb{R}$ or else the subspace topology of A is the discrete topology.

Hint: consider $\inf\{a > 0 : a \in A\}$. [20]